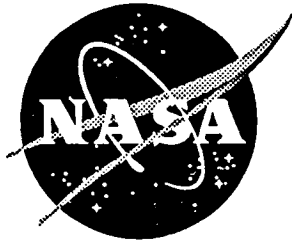


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# Robust Control of Nonlinear Flexible Multibody Systems Using Quaternion Feedback and Dissipative Compensation

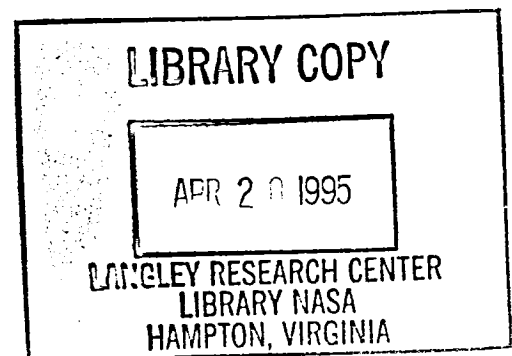
Atul G. Kelkar  
*Old Dominion University, Norfolk, Virginia*

Suresh M. Joshi  
*Langley Research Center, Hampton, Virginia*

*Corrected  
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National Aeronautics and  
Space Administration  
Langley Research Center  
Hampton, Virginia 23681-0001





## Summary

Global asymptotic stability of a class of nonlinear multibody flexible space-structures under dissipative compensation is established. Two cases are considered. The first case allows unlimited nonlinear motion of the entire system and uses quaternion feedback. The second case assumes that the central body motion is in the linear range although the other bodies can undergo unrestricted nonlinear motion. For both cases, the stability is proved to be robust to inherent nonlinearities and modeling uncertainties. Furthermore, for the second case, the stability is also shown to be robust to certain actuator and sensor nonlinearities. The stability proofs use the Lyapunov approach and exploit the inherent passivity of such systems. The results are applicable to a wide class of systems, including flexible space-structures with articulated flexible appendages.

## Introduction

Many space missions envisioned for the future will require flexible multibody space systems such as space platforms with multiple articulated payloads, and space-based manipulators used for satellite assembly and servicing. Such systems are expected to have significant flexibility in the structural members as well as joints. Control systems design for such systems is a difficult problem because of the highly nonlinear dynamics, large number of significant elastic modes with low inherent damping, and uncertainties in the mathematical model. The literature discussed below contains a number of important stability results for certain subclasses of this problem; e.g., linear flexible structures, nonlinear multibody rigid structures, and most recently, multibody flexible structures. Under certain conditions, the input-output maps for such systems can be shown to be "passive" [1]. The Lyapunov and passivity approaches are used in [2] to demonstrate global asymptotic stability of linear flexible space structures (with no articulated appendages) for a class of dissipative compensators. The stability properties were shown to be robust to first-order actuator dynamics and certain actuator/sensor nonlinearities. Multibody rigid structures comprise another class of systems for which stability results have been

advanced. Ideally, subject to certain restrictions, these systems can be categorized as “natural systems” [3]. Such systems are known to exhibit global asymptotic stability under proportional-and-derivative (PD) control. Upon recognition that rigid manipulators belong to the class of natural systems, a number of researchers [4], [5], [6], [7] etc., have established global asymptotic stability of terrestrial rigid manipulators employing PD control with gravity compensation. Stability of tracking controllers was investigated in [8] and [9] for rigid manipulators. In [10], an extension of the results of [9] to the exponentially stable tracking control for flexible multilink manipulators, local to the desired trajectory, was obtained. Lyapunov stability of multilink flexible systems was addressed in [11]. However, the global asymptotic stability for nonlinear, multilink, flexible space-structures has not been addressed in the literature, and that is the subject of this paper.

We consider a complete nonlinear rotational dynamic model of a multibody flexible spacecraft which is assumed to have a branched geometry, i.e., it has a central flexible body to which various flexible appendage bodies are attached (Figure 1). Global asymptotic stability of such systems controlled by dissipative controllers is proved. In many applications the central body has a large mass and moments of inertia as compared to any other appendage bodies. For this case, the effects of realistic nonlinearities in the actuators and sensors are investigated when the central body is in the attitude-hold configuration. The proofs given use the Lyapunov approach. For systems with linear actuators and sensors, the stability proof by Lyapunov’s method can take a simpler form if the Work-Energy Rate principle [11] is used. However, since the Work-Energy Rate principle is applicable only when the system is holonomic and scleronomic in nature, we have used a more direct approach in evaluating the time derivative of the Lyapunov function so that the results are more general.

### Symbols

$B$	control influence matrix
$C$	Coriolis and centrifugal force matrix

$D$	damping matrix
$\tilde{D}$	structural damping matrix
$G_p, \bar{G}_p$	position gain matrices
$G_r$	rate gain matrix
$K$	stiffness matrix of the system
$\tilde{K}$	stiffness matrix for the flexible degrees of freedom
$k$	number of rigid body degrees of freedom
$L$	Lagrangian of the system
$M(p)$	mass-inertia matrix of the system
$n$	number of total degrees of freedom
$p, \hat{p}$	generalized coordinate vectors
$\tilde{p}$	vector of rigid body coordinates
$q$	vector of flexural coordinates
$S$	skew-symmetric matrix
$u$	vector of control input
$V$	Lyapunov function candidate
$x, \bar{x}$	state vectors
$y_p$	position output
$y_r$	rate output
$z$	state vector
$\alpha$	the quaternion
$\alpha_i$	$i$ th component of quaternion
$\bar{\alpha}$	vector part of quaternion
$\hat{\alpha}_i$	$i$ th component of unit vector along eigen axis
$\beta$	scalar defined as $(\alpha_4 - 1)$
$\gamma$	integral of $\omega$
$\eta$	Euler angle vector

$\psi_{ai}$	actuator nonlinearity ( $i$ th loop)
$\psi_{pi}$	position sensor nonlinearity ( $i$ th loop)
$\psi_{ri}$	rate sensor nonlinearity ( $i$ th loop)
$\omega$	angular velocity vector for the central body
$\omega \times$	skew-symmetric matrix of the components of $\omega$
$\theta$	vector of rotational degrees of freedom between rigid bodies

### Mathematical Model

The class of systems considered here have a branched configuration as shown in Figure 1. Each branch by itself could be a serial multibody structure. For the sake of simplicity, and without loss of generality, we will consider a spacecraft with only one such branch (Figure 2) where each appendage body has one degree of freedom (hinge) with respect to the previous body in the chain. The results obtained in this paper, however, will also be applicable to the general case with multiple branches. Consider the spacecraft consisting of a central flexible body and a chain of  $(k - 3)$  flexible links. The central body has three rigid rotational degrees of freedom, and each link is connected by one rotational degree of freedom to the neighboring link. That means there are  $k$  rigid body degrees of freedom. The Lagrangian for the system under consideration can be given by

$$L = \dot{p}^T M(p) \dot{p} - q^T \tilde{K} q \quad (1)$$

where,  $\dot{p} = \{\omega^T, \dot{\theta}^T, \dot{q}^T\}^T$ ;  $\omega$  is the  $3 \times 1$  inertial angular velocity vector for the central body;  $\theta = (\theta_1, \theta_2, \dots, \theta_{(k-3)})^T$ , where  $\theta_i$  denotes the joint angle for the  $i$ th joint expressed in body-fixed coordinates;  $q$  is the  $(n - k)$  vector of flexible degrees of freedom (modal amplitudes);  $M(p) = M^T(p) > 0$  is the configuration-dependent mass-inertia matrix, and  $\tilde{K}$  is the symmetric positive definite stiffness matrix related to the flexible degrees of freedom. Using the Lagrangian (1) the following equations of motion are obtained. The details of the derivation of math model can be found in [13].

$$M(p)\ddot{p} + C(p, \dot{p})\dot{p} + D\dot{p} + Kp = B^T u \quad (2)$$

where  $\{p\} = \{\gamma^T, \theta^T, q^T\}^T$ , and  $\dot{\gamma} = \omega$ .  $C(p, \dot{p})$  corresponds to Coriolis and centrifugal forces;  $D$  is the symmetric, positive semidefinite damping matrix;  $B = [I_{k \times k} \quad 0_{k \times (n-k)}]$  is the control influence matrix and  $u$  is the  $k$ -vector of applied torques. The first three components of  $u$  are the torques applied to the central body by attitude control actuators (one about each body-fixed axis), and the remaining components are the torques applied at the  $(k - 3)$  joints.  $K$  and  $D$  are symmetric, positive semidefinite stiffness and damping matrices:

$$K = \begin{bmatrix} 0_{k \times k} & 0_{k \times (n-k)} \\ 0_{(n-k) \times k} & \tilde{K}_{(n-k) \times (n-k)} \end{bmatrix} \quad D = \begin{bmatrix} 0_{k \times k} & 0_{k \times (n-k)} \\ 0_{(n-k) \times k} & \tilde{D}_{(n-k) \times (n-k)} \end{bmatrix} \quad (3)$$

where  $\tilde{K}$  and  $\tilde{D}$  are symmetric positive definite. The angular measurements for the central body are Euler angles (not the vector  $\gamma$ ), whereas remaining angular measurements between bodies are relative angles. One important inherent property (which we shall call “property  $S$ ”) of such systems that is crucial to the stability results to be presented is given below.

**Property  $S$ :** For the system represented by equation (2), the matrix  $(\frac{1}{2}\dot{M} - C)$  is skew-symmetric.

**Outline of Proof:** Using the indicial notation, the  $k, j$ -th element of  $C(p, \dot{p})$  is defined as

$$c_{kj} = \sum_{i=1}^n c_{ijk}(p) \dot{p}_i = \sum_{i=1}^n \frac{1}{2} \left\{ \frac{\partial M_{kj}}{\partial p_i} + \frac{\partial M_{ki}}{\partial p_j} - \frac{\partial M_{ij}}{\partial p_k} \right\} \dot{p}_i$$

Similarly, the  $kj$ -th component of the time derivative of the inertia matrix,  $\dot{M}(p)$ , is given by the chain rule as

$$\dot{M}_{kj} = \sum_{i=1}^n \frac{\partial M_{kj}}{\partial p_i} \dot{p}_i$$

Now if we define the matrix  $S = (\frac{1}{2}\dot{M} - C)$ , then the  $k, j$ -th element of  $S$  is given by

$$\begin{aligned} S_{kj} &= (\frac{1}{2}\dot{M}_{kj} - C_{kj}) \\ &= \frac{1}{2} \sum_{i=1}^n \left[ \frac{\partial M_{kj}}{\partial p_i} - \left\{ \frac{\partial M_{kj}}{\partial p_i} + \frac{\partial M_{ki}}{\partial p_j} - \frac{\partial M_{ij}}{\partial p_k} \right\} \right] \dot{p}_i \\ &= \frac{1}{2} \sum_{i=1}^n \left[ \frac{\partial M_{ij}}{\partial p_k} - \frac{\partial M_{ki}}{\partial p_j} \right] \dot{p}_i \end{aligned}$$

Since the inertia matrix is symmetric, i.e.,  $M_{ij} = M_{ji}$ , it follows from above equation by

interchanging the indices  $k$  and  $j$  that

$$S_{jk} = -S_{kj}$$

Which means the matrix  $S$  is skew-symmetric. ■

It is assumed that the sensors consist of angular position and rate sensors which are collocated with the torque actuators. The sensor outputs are then given by:

$$y_p = B\hat{p} \quad \text{and} \quad y_r = B\dot{\hat{p}} \quad (4)$$

where  $\hat{p} = (\eta^T, \theta^T, q^T)^T$  wherein  $\eta$  is the Euler angle vector for the central body.  $y_p = (\eta^T, \theta^T)^T$  and  $y_r = (\omega^T, \dot{\theta}^T)^T$  are measured angular position and rate vectors, respectively. It is assumed that the body rate measurements,  $\omega$ , are available via rate gyros.

### Quaternion as a Measure of Attitude

The orientation of a free-floating body can be minimally represented by a 3-dimensional orientation vector. However, this representation is not unique. One minimal representation that is commonly used to represent the attitude is Euler angles. The  $3 \times 1$  Euler angle vector  $\eta$  is given by [14] :  $E(\eta)\dot{\eta} = \omega$ , where  $E(\eta)$  is a  $3 \times 3$  transformation matrix.  $E(\eta)$  becomes singular for certain values of  $\eta$ ; however, it is to be noted that the limitations imposed on the allowable orientations due to this singularity are purely mathematical in nature and have no physical significance. The problem of singularity in 3-parameter representation of attitude has been studied in detail in the literature. An effective way of overcoming the singularity problem is to use the quaternion formulation (see [15]- [17]).

The unit quaternion  $\alpha$  is defined as follows.

$$\alpha = \{\bar{\alpha}^T, \alpha_4\}^T, \quad \bar{\alpha} = \begin{bmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_2 \\ \hat{\alpha}_3 \end{bmatrix} \sin(\frac{\phi}{2}), \quad \alpha_4 = \cos(\frac{\phi}{2}) \quad (5)$$

$\hat{\alpha} = (\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3)^T$  is the unit vector along the eigen-axis of rotation and  $\phi$  is the magnitude of rotation. The quaternion is also subjected to the norm constraint:

$$\bar{\alpha}^T \bar{\alpha} + \alpha_4^2 = 1 \quad (6)$$



It can be also shown [18] that the quaternion obeys the following kinematic differential equations.

$$\dot{\bar{\alpha}} = \frac{1}{2}(\omega \times \bar{\alpha} + \alpha_4 \omega) \quad (7)$$

$$\dot{\alpha}_4 = -\frac{1}{2}\omega^T \bar{\alpha} \quad (8)$$

The attitude control of a single-body rigid spacecraft using a quaternion feedback has been thoroughly investigated [12], [15-17]. We shall use quaternion representation for the central body attitude. The quaternion can be computed [18] using Euler angle measurements (Eq. 4).

Defining  $\beta = (\alpha_4 - 1)$  and denoting  $\dot{p} = z$ , equations (2), (7), and (8) can be rewritten as:

$$M\dot{z} + Cz + Dz + K\{0_{1 \times 3}, \theta^T, q^T\}^T = B^T u \quad (9)$$

$$\dot{\bar{\alpha}} = \frac{1}{2}(\omega \times \bar{\alpha} + (\beta + 1)\omega) \quad (10)$$

$$\dot{\beta} = -\frac{1}{2}\omega^T \bar{\alpha} \quad (11)$$

In equation (9) the matrices  $M$  and  $C$  are functions of  $p$ , and  $(p, \dot{p})$ , respectively. It is to be noted that the first three elements of  $p$  associated with the orientation of central body can be fully described by the unit quaternion. Hence,  $M$  and  $C$  are implicit functions of  $\alpha$ , and therefore, the system represented by equations (9)-(11) is time-invariant and can be expressed in the state-space form as follows:

$$\dot{x} = f(x, u) \quad (12)$$

where  $x = (\bar{\alpha}^T, \beta, \theta^T, q^T, z^T)^T$ . Note that the dimension of  $x$  is  $(2n + 1)$ , which is one more than the dimension of system in (2). However, one constraint (Eq. 6) is now present. It can be easily verified from (6)-(8) that the constraint (6) is satisfied for all  $t > 0$  if it is satisfied at  $t = 0$ .

## Stability with Dissipative Control Law

Consider the control law  $u$ , given by:

$$u = -G_p \tilde{p} - G_r y_r \quad (13)$$

where  $\tilde{p} = \{\bar{\alpha}^T, \theta^T\}^T$ . Matrices  $G_p$  and  $G_r$  are symmetric positive definite  $(k \times k)$  matrices and  $G_p$  is given by:

$$G_p = \begin{bmatrix} (1 + \frac{(\beta+1)}{2})G_{p1} & 0_{3 \times (k-3)} \\ 0_{(k-3) \times 3} & G_{p2(k-3) \times (k-3)} \end{bmatrix} \quad (14)$$

Note that eqs. (13) and (14) represent a nonlinear control law. If  $G_p$  and  $G_r$  are symmetric and positive definite, this control law can be shown to render the time-rate of change of the system's energy negative along all trajectories; i.e., it is a *dissipative* control law. The closed-loop equilibrium solution can be obtained by equating all the derivatives to zero in Eqs. (2), (10), and (11), with the input as in (13) and (14). It can be easily verified that the equilibrium solutions of the closed-loop system given by eqs. (12) and (13) are:  $\bar{\alpha} = 0$ ,  $\theta = 0$ ,  $q = 0$ ,  $z = 0$ , and  $\beta = 0$  or  $-2$  (i.e.,  $\alpha_4 = \pm 1$ ). Thus, there appear to be two closed-loop equilibrium points corresponding to  $\beta = 0$  ( $\alpha_4 = 1$ ) and  $\beta = -2$  ( $\alpha_4 = -1$ ) (all the other state variables being zero). However, from Eq. (5),  $\beta = 0$  ( $\alpha_4 = 1$ )  $\Rightarrow \phi = 0$ , and  $\beta = -2$  ( $\alpha_4 = -1$ )  $\Rightarrow \phi = 2\pi$ , i.e., there is only one equilibrium point in the physical space. One objective of the control law is to transfer the state of the system from one orientation (equilibrium) position to another orientation. Without loss of generality, the target orientation can be defined to be the origin ( $x = 0$ ), and the initial orientation, given by  $(\bar{\alpha}(0), \beta(0), \theta(0))$  can always be defined in such a way that  $|\theta_i(0)| \leq \pi$ , and  $-1 \leq \beta(0) \leq 0$ , i.e.,  $0 \leq \alpha_4(0) \leq 1$  (corresponding to  $|\phi| \leq \pi$ ) and  $(\bar{\alpha}(0), \alpha_4(0))$  satisfy Eq. (6).

The following theorem establishes the global asymptotic stability of the physical equilibrium state of the system.

**Theorem 1.** Suppose  $G_{p2(k-3) \times (k-3)}$  and  $G_{r(k \times k)}$  are symmetric and positive definite, and  $G_{p1} = \mu I_3$  where  $\mu > 0$ . Then, the closed-loop system given by equations (12) and (13) is globally asymptotically stable.

**Proof.**

Consider the Lyapunov function

$$V = \frac{1}{2}\dot{p}^T M(p)\dot{p} + \frac{1}{2}q^T \tilde{K}q + \frac{1}{2}\theta^T G_{p2}\theta + \frac{1}{2}\bar{\alpha}^T (G_{p1} + 2\mu I_3)\bar{\alpha} + \mu\beta^2 \quad (15)$$

$V$  is clearly positive definite and radially unbounded with respect to the state vector  $\{\bar{\alpha}^T, \beta, \theta^T, q^T, \dot{p}^T\}^T$  since  $M(p)$ ,  $\tilde{K}$ ,  $G_{p1}$  and  $G_{p2}$  are positive definite symmetric matrices.

Taking the time derivative, we have:

$$\dot{V} = \dot{p}^T M\dot{p} + \frac{1}{2}\dot{p}^T \dot{M}\dot{p} + \dot{q}^T \tilde{K}q + \dot{\theta}^T G_{p2}\theta + \dot{\bar{\alpha}}^T (G_{p1} + 2\mu I_3)\bar{\alpha} + 2\mu\beta\dot{\beta} \quad (16)$$

Using (2), (4), (10), (11) and (14), we get:

$$\dot{V} = \dot{p}^T B^T u + \dot{p}^T \left( \frac{1}{2}\dot{M} - C \right) \dot{p} - \dot{p}^T D\dot{p} - \dot{p}^T Kp + \dot{q}^T \tilde{K}q + \dot{\theta}^T G_{p2}\theta + \frac{1}{2}(\Omega\bar{\alpha})^T G_{p1}\bar{\alpha} + \frac{1}{2}(\beta+1)\omega^T G_{p1}\bar{\alpha} + \mu\omega^T \bar{\alpha} \quad (17)$$

where  $\Omega = (\omega \times)$  is a skew-symmetric matrix. Substituting for  $u$  and noting that,  $\dot{p}^T Kp = \dot{q}^T \tilde{K}q$ ,  $(\Omega\bar{\alpha})^T G_{p1}\bar{\alpha} = 0$ , and using Property  $\mathcal{S}$  of the system, we obtain

$$\dot{V} = -\dot{p}^T (D + B^T G_r B) \dot{p} - (B\dot{p})^T G_p \tilde{p} + \frac{1}{2}(\beta+1)\omega^T G_{p1}\bar{\alpha} + \mu\omega^T \bar{\alpha} + \dot{\theta}^T G_{p2}\theta \quad (18)$$

Note that  $(B\dot{p})^T G_p \tilde{p} = \frac{1}{2}(\beta+1)\omega^T G_{p1}\bar{\alpha} + \mu\omega^T \bar{\alpha} + \dot{\theta}^T G_{p2}\theta$ . After several cancellations, we get

$$\dot{V} = -\dot{p}^T (D + B^T G_r B) \dot{p} \quad (19)$$

Since  $(D + B^T G_r B)$  is a positive definite symmetric matrix,  $\dot{V} \leq 0$ , i.e.,  $\dot{V}$  is negative semidefinite, and  $\dot{V} = 0 \Rightarrow \dot{p} = 0 \Rightarrow \ddot{p} = 0$ . Substituting in the closed-loop equation we get

$$-B^T G_p \tilde{p} = \begin{bmatrix} -G_p \tilde{p} \\ 0_{(n-k) \times 1} \end{bmatrix} = \begin{bmatrix} 0_{k \times 1} \\ \tilde{K}q \end{bmatrix} \quad (20)$$

$\Rightarrow \tilde{p} = 0$ , and  $q = 0$ , i.e.,  $\bar{\alpha} = 0$ ,  $\theta = 0$ , and  $\beta = 0$  or  $-2$ . Thus,  $\dot{V} < 0$  along all trajectories, and  $\dot{V} = 0$  at the two equilibrium points. Therefore, if the system's initial condition lies anywhere in the state space except at the equilibrium point corresponding to  $\beta = -2$ , then the trajectory will asymptotically approach the origin, i.e.,  $x = 0$ ; and if the system is at

the equilibrium point corresponding to  $\beta = -2$  at  $t = 0$  then it will stay there for all  $t > 0$ . However, consistent with the previous discussion, the two equilibrium points in the state space represent the same equilibrium point in the physical space; hence it can be concluded that the system is globally asymptotically stable. ■

### **A Special Case:**

Consider a special case where the central body attitude motion is small. This can occur in many realistic situations. For example, in the case of a space station-based or Shuttle-based manipulator, the inertia of the base (central body) is much larger than that of any manipulator link or payload. In such cases the rotational motion of the base can be assumed to be in the linear region, although the payloads (or links) attached to it can undergo large rotational and translational motions and nonlinear dynamic loading due to Coriolis and centripetal accelerations. For this case, the attitude of the central body is simply the integral of the inertial angular velocity and the use of quaternions is not necessary. The equations of motion (2) can now be expressed in the state-space form simply as:

$$\dot{\bar{x}} = g(\bar{x}, u) \quad (21)$$

where  $\bar{x} = (p^T, \dot{p}^T)^T$ .

The dissipative control law  $u$  is now given by:

$$u = -\bar{G}_p y_p - G_r y_r \quad (22)$$

where,  $\bar{G}_p$  is symmetric positive definite  $(k \times k)$  matrix,

$$y_p = Bp \quad \text{and} \quad y_r = B\dot{p} \quad (23)$$

$y_p$  and  $y_r$  are measured angular position and rate vectors.

**Theorem 2.** Suppose  $\bar{G}_{p_{k \times k}}$  and  $G_{r_{k \times k}}$  are symmetric and positive definite. Then, the closed-loop system given by equations (21), (22) and (23) is globally asymptotically stable.

### **Proof.**

Consider the Lyapunov function

$$V = \frac{1}{2}\dot{p}^T M(p)\dot{p} + \frac{1}{2}p^T (K + B^T \overline{G}_p B)p \quad (24)$$

$V$  is clearly positive definite since  $M(p)$  and  $(K + B^T \overline{G}_p B)$  are positive definite symmetric matrices. Taking the time derivative, letting  $\overline{K} = (K + B^T \overline{G}_p B)$ , and simplifying, we get

$$\dot{V} = \dot{p}^T \left( \frac{1}{2} \dot{M} - C \right) \dot{p} - \dot{p}^T \overline{K} p + \dot{p}^T \overline{K} p - \dot{p}^T (D + B^T G_r B) \dot{p} \quad (25)$$

Again, using Property  $\mathcal{S}$ , we get,  $\dot{p}^T (\frac{1}{2} \dot{M} - C) \dot{p} = 0$ , and after some cancellations, we obtain

$$\dot{V} = -\dot{p}^T (D + B^T G_r B) \dot{p} \quad (26)$$

Since  $(D + B^T G_r B)$  is the positive definite symmetric matrix,  $\dot{V} \leq 0$ , i.e.,  $\dot{V}$  is negative semidefinite in  $p$  and  $\dot{p}$  and  $\dot{V} = 0 \Rightarrow \dot{p} = 0 \Rightarrow \ddot{p} = 0$ . Substituting in the closed-loop equation we get

$$(K + B^T \overline{G}_p B)p = 0 \quad \Rightarrow p = 0 \quad (27)$$

Thus,  $\dot{V}$  is not zero along any trajectories; then, by LaSalle's theorem, the system is globally asymptotically stable. ■

The significance of the two results presented above is that *any nonlinear multibody system belonging to these classes can be robustly stabilized with dissipative control laws*. In the case of manipulators, this means that one can accomplish any terminal angular position from any initial position with *guaranteed asymptotic stability*.

### **Robustness to Actuator/Sensor Nonlinearities**

Theorem 2 proves global asymptotic stability for the practically important case where the central body motion is in the linear range and the other bodies undergo nonlinear motion. It assumes linear actuators and sensors. In practice, however, the actuators and sensors have nonlinearities. The following theorem extends the results of [2] to the case of nonlinear flexible multibody systems. That is, the robust stability property of the

dissipative controller is proved to hold in the presence of a wide class of actuator/sensor nonlinearities.

Let  $\psi_{ai}(\cdot)$ ,  $\psi_{pi}(\cdot)$ , and  $\psi_{ri}(\cdot)$  denote the nonlinearities in the  $i$ th actuator, position sensor, and rate sensor channels, respectively. Assuming  $\bar{G}_p$  and  $G_r$  are diagonal, the actual input is given by:

$$u_i = \psi_{ai}[-\bar{G}_{pi}\psi_{pi}(y_{pi}) - G_{ri}\psi_{ri}(y_{ri})] \quad (i = 1, 2, \dots, k) \quad (28)$$

We assume that  $\psi_{pi}$ ,  $\psi_{ai}$  and  $\psi_{ri}$  ( $i = 1, 2, \dots, k$ ) are continuous single-valued functions:  $\mathbf{R} \rightarrow \mathbf{R}$ . [A function  $\psi(\nu)$  is said to belong to the  $(0, \infty)$  sector (Figure 3) if  $\psi(0) = 0$  and  $\nu\psi(\nu) > 0$  for  $\nu \neq 0$ :  $\psi$  is said to belong to the  $[0, \infty)$  sector if  $\nu\psi(\nu) \geq 0$ ]. The following theorem gives sufficient conditions for stability.

**Theorem 3.** Consider the closed-loop system given by (21), (22), (23), and (28), where  $\bar{G}_p$  and  $G_r$  are diagonal with positive entries. Suppose (for  $i = 1, 2, \dots, k$ )  $\psi_{ai}$ ,  $\psi_{pi}$ , and  $\psi_{ri}$  are single-valued, time invariant continuous functions belonging to the  $(0, \infty)$  sector and  $\psi_{ai}$  are monotonically nondecreasing. Under these conditions, the closed-loop system is globally asymptotically stable.

**Proof.**

(The proof closely follows [2].) Let  $w = -y_p$  ( $k$ -vector). Define

$$\bar{\psi}_{pi}(\nu) = -\psi_{pi}(-\nu) \quad (29)$$

$$\bar{\psi}_{ri}(\nu) = -\psi_{ri}(-\nu) \quad (30)$$

If  $\psi_{pi}$ ,  $\psi_{ri} \in (0, \infty)$  or  $[0, \infty)$  sector then  $\bar{\psi}_{pi}$ ,  $\bar{\psi}_{ri}$  also belong to the same sector. Now, consider the following Lur -Postnikov Lyapunov function :

$$V = \frac{1}{2}\dot{p}^T M(p)\dot{p} + \frac{1}{2}q^T \tilde{K}q + \sum_{i=1}^k \int_0^{w_i} \psi_{ai}\{\bar{G}_{pi}\bar{\psi}_{pi}(\nu)\}d\nu \quad (31)$$

where,  $\tilde{K}$  is the symmetric positive definite part of  $K$ . Taking the time derivative and using (2),

$$\dot{V} = \dot{p}^T [B^T u - C\dot{p} - D\dot{p} - Kp] + \frac{1}{2}\dot{p}^T \dot{M}\dot{p} + \sum_{i=1}^k \dot{w}_i \psi_{ai}\{\bar{G}_{pi}\bar{\psi}_{pi}(w_i)\} + q^T \tilde{K}q \quad (32)$$

Upon several cancellations and using Property  $\mathcal{S}$ ,

$$\dot{V} = \sum_{i=1}^k u_i y_{ri} - \dot{q}^T \tilde{D} \dot{q} + \sum_{i=1}^k \dot{w}_i \psi_{ai} \{ \bar{G}_{pi} \bar{\psi}_{pi}(w_i) \} \quad (33)$$

where, matrix  $\tilde{D}$  is the positive definite part of  $D$ .

$$\dot{V} = -\dot{q}^T \tilde{D} \dot{q} - \sum_{i=1}^k \dot{w}_i (\psi_{ai} [G_{ri} \bar{\psi}_{ri}(\dot{w}_i) + \bar{G}_{pi} \bar{\psi}_{pi}(w_i)] - \psi_{ai} [\bar{G}_{pi} \bar{\psi}_{pi}(w_i)]) \quad (34)$$

If  $\psi_{ai}$  are monotonic nondecreasing and  $\psi_{ri}$  belong to the  $(0, \infty)$  sector,  $\dot{V} \leq 0$ , and it can be concluded that the system is at least Lyapunov-stable. Now we will prove that in fact the system is globally asymptotically stable. First, let us consider a special case when  $\psi_{ai}$  are monotonic increasing. Then  $\dot{V} \leq -\dot{q}^T \tilde{D} \dot{q}$ , and  $\dot{V} = 0$  only when  $\dot{q} = 0$  and  $\dot{w} = 0$ , which implies  $\dot{p} = 0 \Rightarrow \ddot{p} = 0$ . Substituting in the closed-loop equation,

$$Kp = B^T \psi_a [-\bar{G}_p \psi_p(y_p)] \quad (35)$$

$$\begin{bmatrix} 0 \\ \tilde{K}q \end{bmatrix} = \begin{bmatrix} \psi_a \{ -\bar{G}_p \psi_p(y_p) \} \\ 0 \end{bmatrix} \quad (36)$$

$$\Rightarrow \psi_a [-\bar{G}_p \psi_p(y_p)] = 0, \quad \text{and} \quad q = 0$$

If  $\psi_{pi}$  belong to the  $(0, \infty)$  sector,  $\psi_{ai}(\nu) = \psi_{pi}(\nu) = 0$  only when  $\nu = 0$ . Therefore,  $y_p = 0$ . Thus,  $\dot{V} = 0$  only at the origin, and the system is globally asymptotically stable.

In the case when actuator nonlinearities are of the monotonic nondecreasing type (such as saturation nonlinearity),  $\dot{V}$  can be 0 even if  $\dot{w} \neq 0$ . Figure 4 shows a monotonically nondecreasing nonlinearity. However, we will show that every system trajectory along which  $\dot{V} \equiv 0$ , has to go to the origin asymptotically. When  $\dot{w} \neq 0$ ,  $\dot{V} \equiv 0$  only when all actuators are locally saturated. Then, from the equations of motion, it means that system trajectories will go unbounded which is not possible since we have already proved that the system is Lyapunov-stable. Hence, system trajectories have to approach the origin asymptotically, and the system is globally asymptotically stable. ■

## Concluding Remarks

Stability of a class of nonlinear multibody flexible space systems was considered using a class of dissipative control laws. It was shown that robust global asymptotic stability can be obtained using a nonlinear feedback of the central body quaternion angles, relative body angles, and angular velocities. For the practically important special case wherein the central body motion is in the linear range, it was shown that global asymptotic stability is obtained with a linear dissipative control law. Furthermore, it was shown that the robust stability is preserved in the presense of a wide class of actuator and sensor nonlinearities. All the stability results presented are valid in spite of modeling errors and parametric uncertainties. The results have a significant practical value since the mathematical models of such systems usually have substantial inaccuracies, and the actuation and sensing devices have nonlinearities.

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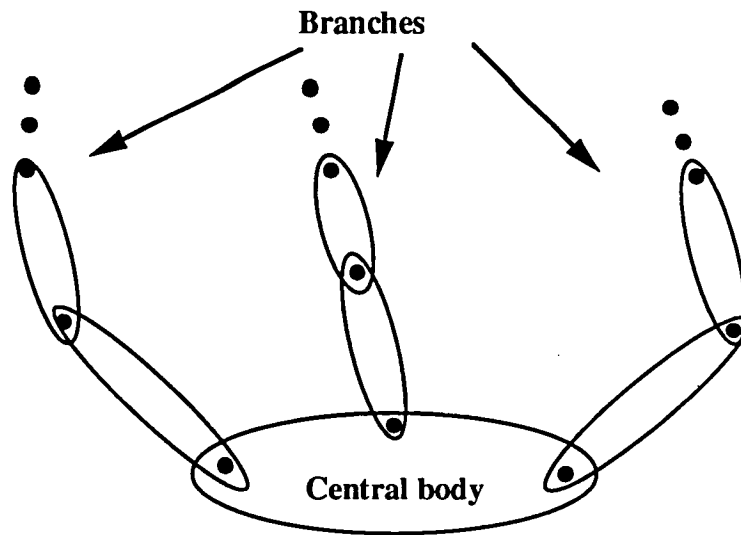
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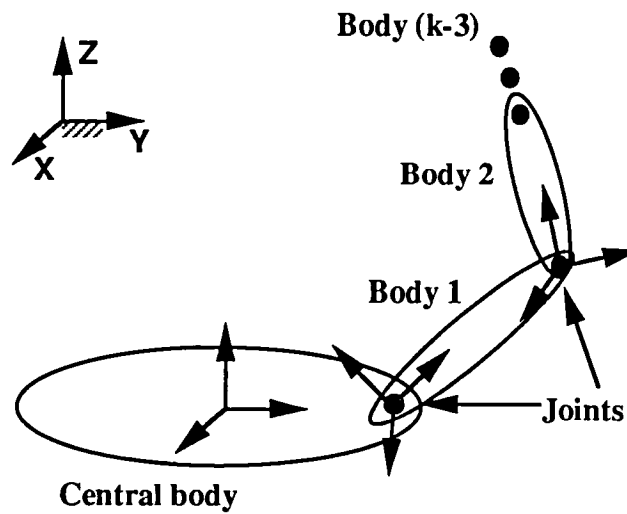
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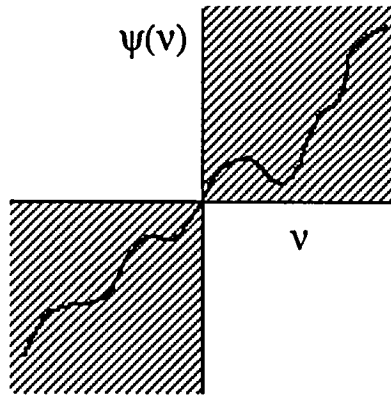
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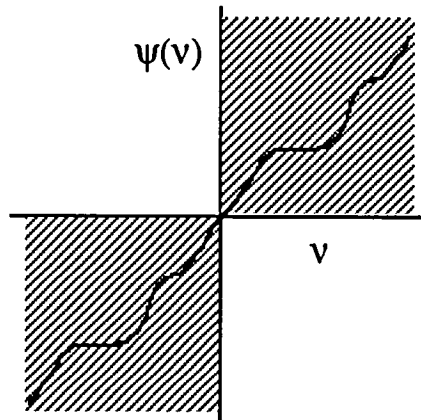
**Figure 1. Multibody system**



**Figure 2. Multibody system with a single chain**



**Figure 3. Nonlinearity belonging to  $(0, \infty)$  sector**



**Figure 4. Monotonically non-decreasing nonlinearity**

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